TESTING EQUALITY OF DEPENDENT CORRELATION COEFFICIENTS

By

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To my parents,

to Beom, my husband,

and

to Hockewn and Tongkewn, my beloved children.

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The t-test for testing $H_o: \rho_{jk} = 0$ was adapted to test H_o: $\rho_{12} = \rho_{13}$ by demonstrating that $H_o: \rho_{12} = \rho_{13}$ is equivalent to $H_o': \rho_{1D} = 0$, where $D = z_2 - z_3 = 0$ and z_i is a standardized variable. The performance of the proposed test was compared to Williams' procedure and three procedures based on the Fisher Z transformation under the various combinations of ρ_{12}, ρ_{13}, and ρ_{23}, nominal alpha levels of .01, .05, and .10, and sample sizes of 30, 65, and 100. Simulation methods were used in the comparisons.

Results were interpreted in terms of Type I error rates and statistical power to test the equality of dependent correlation coefficients. The performance of the proposed procedure was inferior to other procedures in most cases. The alternatives exhibited good control of the Type I error rate. There were minimal differences in power among the alternatives.

CHAPTER 1

INTRODUCTION

A common purpose in educational research is the comparison of two correlation coefficients calculated by using data from the same individuals. These two correlations are not, in general, independent of each other. In testing equality of two correlation coefficients, there are two distinct cases to be considered: two correlations with a single variate in common (H_o: $\rho_{12} = \rho_{13}$) and two correlation coefficients from four distict variates (H_o: $\rho_{12} = \rho_{34}$). In this thesis, tests of H_o: $\rho_{12} = \rho_{13}$ were considered.

Some Test Statistics for Testing H_o: $\rho_{12} = \rho_{13}$

Statistical tests for the hypothesis H_o: $\rho_{12} = \rho_{13}$ have been considered by many authors, including Hotelling (1940), Williams (1959), and Dunn and Clark (1969). Hotelling's statistic is

$$T_1 = (n - 3)^{1/2}(r_{12} - r_{13})(1 + r_{23})^{1/2}(2|R|)^{-1/2}$$

where

$$|R| = (1 - r_{12}^2 - r_{13}^2 - r_{23}^2) + (2r_{12}r_{13}r_{23})$$

is the determinant of the 3 x 3 correlation matrix. The critical value for T_1 is t with n - 3 degrees of freedom. Dunn and Clark (1969) reported that T_1 is actually a test of

$H_o: \rho_{12}(1 + m\rho_{23}) = \rho_{13}(m + \rho_{23})$ where $m = (S_{33}\sigma_{22}/S_{22}\sigma_{33})^{1/2}$. Asymptotically m goes to one. Therefore T_1 is asymptotically a test of H_o: $\rho_{12} = \rho_{13}$. In small samples, the Type I error rate of T_1 can be very high even when H_o: $\rho_{12} = \rho_{13}$ is true. To overcome this difficulty, Williams (1959) proposed a modification of Hotelling's T_1 statistic. Williams' statistic is

$$T_2 = (r_{12} - r_{13})\left[\frac{(n - 1)(1 + r_{23})}{2\frac{n - 1}{n - 3}|R| + \bar{r}^2(1 - r_{23})^3}\right]^{1/2}$$

where $\bar{r} = \frac{1}{2}(r_{12} + r_{13})$. The critical value for T_2 is t with n - 3 degrees of freedom.

An alternative to the preceding test statistics is

$$Z_1 = n^{1/2}(r_{12} - r_{13})/(\hat{\psi}_{12}^2 + \hat{\psi}_{13}^2 - 2\hat{\psi}_{12,13})^{1/2}$$

where, under the assumption of trivariate normality, $\hat{\psi}_{jk}^2$ is the estimated asymptotic variance of r_{jk} and $\hat{\psi}_{12,13}$ is the estimated asymptotic covariance of r_{12} and r_{13}. Specifically

$$\hat{\psi}_{jk}^2 = (1 - r_{jk}^2)^2$$

and

$$\hat{\psi}_{12,13} = r_{23}(1 - r_{12}^2 - r_{13}^2) - (1/2)(r_{12}r_{13})(1 - r_{12}^2 - r_{13}^2 - r_{23}^2).$$

Dunn and Clark (1971) developed Z_1^*, a statistic that uses the Fisher's z transformation of the correlations r_{12}, r_{13}, and r_{23}. The Dunn and Clark statistic is

$$Z_1^* = (n - 3)^{1/2}(z_{12} - z_{13})(2 - 2S_{12,13})^{-1/2}$$

where

$$z_{jk} = \tfrac{1}{2}\ln(1 + r_{jk})/(1 - r_{jk})$$

and

$$S_{12.13} = \hat{\psi}_{12.13} / [(1 - r_{12}^2)(1 - r_{13}^2)]$$

The critical value is the z critical value. Results in Brown and Shapiro (1986), Isserlis (1916) and Steiger and Hakstian (1982) can be used to develop a test in which the distributional assumption is relaxed to sampling from a multivariate distribution with finite fourth order moments.

Neil and Dunn (1975) introduced the likelihood ratio test of H_o: $\rho_{12} = \rho_{13}$ and a variant of Z_1^*. This variant, subsequently called $\overline{Z}_1^*$ by Steiger (1980), is

$$\overline{Z}_1^* = (n - 3)^{1/2}(z_{12} - z_{13})/(2 - 2\overline{S}_{12.13})^{1/2}$$

The critical value is the z critical value. The statistic $\overline{S}_{12.13}$ is $S_{12.13}$ with $\frac{1}{2}(r_{12} + r_{13})$ substituted for r_{12} and for r_{13}.

Wolfe (1976) proposed adapting the t statistic for testing H_o: $\rho_{jk} = 0$ to testing H_o: $\rho_{12} = \rho_{13}$. That is, by setting $D = x_3 - x_2$, he demonstrated that H_o: $\rho_{12} = \rho_{13}$ is equivalent to H_o': $\rho_{1D} = 0$, when $\sigma_2^2 = \sigma_3^2$. The t statistic is

$$t = r_{1D}\,[(n - 2)/(1 - r_{1D}^2)]^{1/2}$$

Wolfe's procedure can be extended to the case in which $a = \sigma_3/\sigma_2$ is known. Then H_o: $\rho_{12} = \rho_{13}$ is tested by using the correlation between x_1 and $x_3 - ax_2$ in the usual t test. Choi (1977) extended Wolfe's procedure to test H_o:$\rho_{12} = \rho_{13} = \ldots = \rho_{1k}$ and developed a nonparametric procedure for testing the same hypothesis.

Recently Meng, Rosenthal, and Rubin (1992) developed the statistic

$$Z = (z_{12} - z_{13})\left[\frac{n - 3}{2(1 - r_{23})h}\right]^{1/2}$$

where

$$h = \frac{1 - f(r_{12}^2 + r_{13}^2)/2}{1 - (r_{12}^2 + r_{13}^2)/2}$$

and

$$f = \frac{1 - r_{23}}{2[1 - (r_{12}^2 + r_{13}^2)/2]}$$

They cite computational ease as an advantage of Z.

Review of Related Literature

Dunn and Clark (1971) compared Type I error rates and power of T_1, T_2, and Z_1^* as well as several other statistics. One of these was obtained by substituting population correlations in the denominator of Z_1^*. For this alternative, power can be calculated rather than estimated. Dunn and Clark refer to the calculated power as asymptotic power. They found that with moderate sample sizes (n = 26) the power of Z_1^* was close to the asymptotic power. They also found the power of T_1 was somewhat larger than that of the asymptotic power, whereas the power of T_2 was somewhat smaller. Actual Type I error rates were close to nominal levels for T_1, T_2, and Z_1^*. They concluded that T_2 should be used to test H_o: $\rho_{12} = \rho_{13}$.

Neill and Dunn (1975) compared Type I error rates and power for the likelihood ratio test, T_1, T_2, Z_1^*, $\overline{Z}_1^*$ and several

other statistics. They found that the Type I error rate for T_1 can be excessive; in some conditions it approached .9 for an $\alpha = .05$ test. The Type I error rate of the likelihood ratio test was also unsatisfactory. The tests T_2, Z_1^*, and $\overline{Z}_1^*$ typically had satisfactory Type I error rates. However, for an $\alpha = .05$ test the Type I error rate for Z_1^* was .095 in one condition. Neil and Dunn reported that $\overline{Z}_1^*$ is typically more powerful than either T_2 or Z_1^*. Based on the findings that (a) in one condition T_2 had better power than Z_1^* and (b) in another condition Z_1^* had a .095 Type I error rate for an $\alpha = .05$ test, Neil and Dunn concluded that T_2 is the best overall test.

Summarizing the literature on T_1, T_2, Z_1, Z_1^*, and $\overline{Z}_1^*$, Steiger (1980) dismissed T_1 because it can have excessively high Type I error rates. He also reported that T_2, Z_1^*, and $\overline{Z}_1^*$ are superior to Z_1 in terms of Type I error. Steiger did not state a preference among T_2, Z_1^*, and $\overline{Z}_1^*$.

Choi (1977) investigated the power of Wolfe's (1975) generalized test and Choi's nonparametric test for testing H_o: $\rho_{12} = \rho_{13}$. Choi found that under normality, the parametric procedure is slightly more powerful than the nonparametric procedure.

Purpose of the Study

Hotelling developed T_1, the first test of H_o: $\rho_{12} = \rho_{13}$. However, as mentioned above, in some conditions T_1 almost always rejects the null hypothesis (Steiger, 1980). Meng et

al. (1992) also pointed out that Hotelling's statistic is not appropriate in practice.

Several studies (Dunn and Clark, 1971; Neil and Dunn, 1975) have compared the Type I error rates and power of T_2, Z_1, Z_1^*, and $\overline{Z}_1^*$. Summarizing these studies, Steiger (1980) concluded that T_2, Z_1^*, and $\overline{Z}_1^*$ are superior to Z_1.

Wolfe (1976) developed an alternative to T_2, Z_1^*, and $\overline{Z}_1^*$. Choi (1977) extended Wolfe's test and developed a nonparametric alternative. A problem with Wolfe's test and Choi's parametric extension is that they require knowledge of certain standard deviation ratios. An alternative to Wolfe's test is to estimate the required standard deviation ratio and to use the correlation between x_1 and $x_3 - \hat{a}x_2$ in the usual t statistic in order to test H_o: $\rho_{12} = \rho_{13}$. Using results in Choi, it can be shown that the sample correlation between x_1 and $x_3 - \hat{a}x_2$ is

$$r = (r_{13} - r_{12})/[2(1 - r_{23})]^{1/2}$$

Thus the proposed test statistic is

$$t = r\,[(n - 2)/(1 - r^2)]^{1/2}$$

and the critical value is t with n - 2 degrees of freedom. In the remainder of the thesis the proposed test is referred to as P.

The purpose of the study is to compare Type I error rates and power of T_2, Z_1^*, $\overline{Z}_1^*$, Z, and the proposed statistic.

A Characterization of the Proposed Test

Let $z_i = (x_i - \mu_i)/\sigma_i$. Now

$$\rho(z_1,\ z_2 - z_3) = \frac{\sigma(z_1,\ z_2 - z_3)}{\sigma(z_1)\,\sigma(z_2 - z_3)}$$

By the algebra of variances and covariances of linear combinations

$$\rho(z_1,\ z_2 - z_3) = \frac{\rho_{12} - \rho_{13}}{[2(1 - \rho_{23})]^{1/2}}$$

Thus if H_o: $\rho_{12} = \rho_{13}$ is true, H_o: $\rho(z_1,\ z_2 - z_3) = 0$ is also true and the former can be tested by testing the latter. Let $\hat{z}_i = (x_i - \bar{x}_i)/S_i$. By definition

$$r(\hat{z}_1,\ \hat{z}_2 - \hat{z}_3) = \frac{\Sigma \hat{z}_1(\hat{z}_2 - \hat{z}_3)}{nS(\hat{z}_1)S(\hat{z}_2 - \hat{z}_3)}$$

By expanding the numerator

$$r(\hat{z}_1,\ \hat{z}_2 - \hat{z}_3) = \frac{\Sigma \hat{z}_1\hat{z}_2 - \Sigma \hat{z}_1\hat{z}_3}{nS(\hat{z}_1)S(\hat{z}_2 - \hat{z}_3)}$$

Because $r_{12} = \Sigma \hat{z}_1\hat{z}_2/n$, $r_{13} = \Sigma \hat{z}_1\hat{z}_3/n$ and $S(z_1) = 1$ this fraction can be simplified to

$$r(\hat{z}_1,\ \hat{z}_2 - \hat{z}_3) = \frac{r_{12} - r_{13}}{S(\hat{z}_2 - \hat{z}_3)}$$

By expanding the denominator

$$r(\hat{z}_1,\ \hat{z}_2 - \hat{z}_3) = \frac{r_{12} - r_{13}}{[S^2(\hat{z}_2) + S^2(\hat{z}_3) - 2S(\hat{z}_2\hat{z}_3)]^{1/2}}$$

Because the sample variances of $\hat{z}_2$ and $\hat{z}_3$ are 1, and

$S(\hat{z}_2, \hat{z}_3) = S(\hat{z}_1)S(\hat{z}_2)r_{23}$,

$$r(\hat{z}_1, \hat{z}_2 - \hat{z}_3) = \frac{r_{12} - r_{13}}{[2(1 - r_{23})]^{1/2}}$$

Thus the proposed test is equivalent to using $r(\hat{z}_1, \hat{z}_2 - \hat{z}_3)$ to test H_o: $\rho_{12} = \rho_{13}$.

CHAPTER 2

METHOD

In this study, tests of H_o: $\rho_{12} = \rho_{13}$ were investigated. Specifically, Type I error rates and powers of T_2, Z, Z_1^*, $\overline{Z}_1^*$, and a test proposed in this study were estimated by simulating sample variance-covariance matrices for a trivariate normal distribution. Factors of interest were: (a) sample size, (b) population correlation matrices, and (c) significance level. In this chapter, the design is presented and the simulation method is described.

Design

Sample size. Three levels of sample size were employed: 30, 65, and 100 representing small, moderate, and moderately large correlation studies.

Population correlation matrices. If $\rho_{12} = \rho_{13} = \rho$, the determinant of the population correlation matrix is $(1 - 2\rho^2 - \rho_{23}^2) + (2\rho^2\rho_{23})$. For a trivariate normal distribution, this determinant must be larger than zero. Thus, using this expression, permissible combinations of ρ_{23} and $\rho_{12} = \rho_{13}$ can be determined. Table 1 shows the range of permissible values of $\rho_{12} = \rho_{13}$ for several values of ρ_{23}. Based on the results in Table 1, correlation matrices were selected for the

simulation. These matrices meet conditions that (a) the determinant is larger than zero and (b) the value of ρ_{13} is equal to or larger than the value of ρ_{12} for each value of ρ_{23}. Table 2 shows the combinations of ρ_{12} and ρ_{13} for each value of ρ_{23} included in the simulation. For each of three sample sizes, the 23 cases in which $\rho_{12} = \rho_{13}$ for each value of ρ_{23} were tested to estimate Type I error rates. The conditions with $\rho_{12} < \rho_{13}$ for each value of ρ_{23} were tested to estimate power.

Significance levels. Actual Type I error rates and power were estimated for nominal α = .01, .05, and .10.

Simulation

Generating covariance matrices. Let T be a p x p lower triangular matrix. If $T_{ij} \sim N(0,1)$, for $i \neq j$, and $T_{ii} \sim \chi(n - i + 1)$ where n is the number of observation in a sample, then $TT' \sim W(n - 1, I_p)$, a Wishart distribution with n - 1 degrees of freedom. Thus, $S = (n - 1)^{-1}TT'$ is a covariance matrix for a sample of size n from a population with covariance matrix I. That is, S is a sample covariance matrix for variables that have zero correlation in the population. Pre and post multipling S by M and M' to obtain MSM' yields a sample covariance matrix for a population with covariance matrix MM'. Based on this theory, a sample covariance matrix was generated by using the following steps:

1. For a particular correlation matrix Σ, determine M by a Cholesky decomposition of Σ.

2. Generate S as described above. The variates T_{ij} were

generated by using RANNOR function in SAS; the T_{ii} were generated by using the function RANGAM in SAS.

3. Calculate a sample estimate of Σ by MSM'.

Let $\underline{X}$ denote a (3 x 1) random vector, with elements that have variances equal to one. Then Σ, the covariance matrix for $\underline{X}$, is a correlation matrix. Let $\underline{X}^* = \Delta^*\underline{X}$ where Δ^* is a diagonal matrix. Then $\Sigma^* = \Delta^*\Sigma\Delta^*$ is the covarance matrix for $\underline{X}^*$ and $\underline{X}^*$ has the same correlation matrix as $\underline{X}$ does. There are infinite number of choices for Δ^*. Consequently, for any correlation matrix included in the simulation, there are an infinite number of covariance matrices including the covariance matrix that is equal to the correlation matrix. However, the tests investigated in this study are all functions of correlations and are invariant to transformations such as $\underline{X}^* = \Delta^*\underline{X}$. Consequently, for each correlation matrix it is only necessary to generate sample covariance matrices for the population with covariance matrix Σ, that is with a covariance matrix equal to the correlation matrix.

Evaluating the performance of test statistics. Each condition was replicated 2000 times. For each replication, the data were analyzed by using T_2, Z, Z_1^*, $\overline{Z}_1^*$, and the new test proposed in this paper. The test statistics and the critical values were calulated by using PROC IML in SAS. For each test the proportion of 2000 replications that yielded significant results at α = .10, .05, and .01 was recorded.

Table 1

Permissible Values of $\rho_{12} = \rho_{13}$

ρ_{23}	$\rho_{12} = \rho_{13}$
.8	(-.94, .94)
.6	(-.89, .89)
.4	(-.83, .83)
.2	(-.77, .77)
.0	(-.70, .70)

Table 2

Correlation Combination Included in the Simulation

ρ_{23}	ρ_{12}	ρ_{13}
.0	.0	.0, .2, .4, .6, .8
.0	.2	.2, .4, .6, .8
.0	.4	.4, .6, .8
.0	.6	.6, .8
.2	.0	.0, .2, .4, .6, .8
.2	.2	.2, .4, .6, .8
.2	.4	.4, .6, .8
.2	.6	.6, .8
.4	.0	.0, .2, .4, .6, .8
.4	.2	.2, .4, .6, .8
.4	.4	.4, .6, .8
.4	.6	.6, .8
.4	.8	.8
.6	.0	.0, .2, .4, .6, .8
.6	.2	.2, .4, .6, .8
.6	.4	.4, .6, .8
.6	.6	.6, .8
.6	.8	.8
.8	.0	.0, .2, .4, .6
.8	.2	.2, .4, .6
.8	.4	.4, .6, .8
.8	.6	.6, .8
.8	.8	.8

CHAPTER 3

RESULTS AND DISCUSSION

Type I Error Probabilities

One purpose of this study was to examine the estimated actual alpha levels of five procedures when those procedures were used to test the null hypothesis of equality of correlation coefficients (H_o: $\rho_{12} = \rho_{13}$). Each condition was replicated 2000 times, and the resulting data were analyzed using the five procedures described in Chapter 1. Nominal alpha levels of .01, .05, and .10 were used in each case. For each test and alpha level, the proportion of the 2000 replications resulting in the significant statistics provided an estimate of the actual alpha level. To identify those conditions for which a test did not preserve the actual alpha level, a criterion interval of $\alpha \pm 2[\alpha(1 - \alpha)/2000]^{1/2}$ was used for nominal alpha levels of .01, .05, and .10. When $\alpha = .01$, the criterion interval was (.006,.014); when $\alpha = .05$, the criterion interval was (.04,.06); and when $\alpha = .10$ the interval was (.087,.113). Estimated Type I error rates are reported in Tables 3 through 7 for the five tests; these results are for the 23 cases in which $\rho_{12} = \rho_{13}$ and are for n = 30, 65, and 100 displayed for nominal alpha levels of .01, .05, and .10.

Examination of Table 3 shows that when $\rho_{12} = \rho_{13} = 0$, actual Type I error rates were very similar to the nominal Type I error rate for all tests, values of ρ_{23}, and sample sizes. Consequently almost all estimated actual Type I error rates were in the pre-established criterion intervals.

Inspection of Table 4 shows that when $\rho_{12} = \rho_{13} = .2$, each of the five tests had some estimated actual alpha levels outside the pre-established intervals. However, these aberrant alpha levels were never too distant from the criterion interval limits.

Inspection of Tables 5 through 7 shows that when $\rho_{12} = \rho_{13}$ are .4, .6, or .8, all estimated alpha levels for the proposed test were below the lower limits of the criterion intervals. When the correlation coefficients $\rho_{12} = \rho_{13}$ increased from .4 to .8, aberrant alpha levels for the proposed test were further below the lower limits of the criterion intervals. For the remaining four tests actual Type I error rates were very similar to the nominal alpha levels for all sample sizes and values of ρ_{23}.

From Tables 3 through 7, it is apparent that while the performance of the proposed statistic turned out to be very poor, the performance of the tests T_2, Z_1^*, $\overline{Z}_1^*$, and Z was quite good. Also, the Z test had as small a number of aberrant alpha levels as did the T_2, Z_1^*, and $\overline{Z}_1^*$ tests.

Power

The second purpose of this study was to compare the statistical power for the five tests. Values selected for $\rho_{13} - \rho_{12}$ were .2, .4, .6, and .8 for each combination of ρ_{23} (0, .2, .4, .6, .8) and sample sizes (30, 65, and 100). For each of the 2000 replications of each condition the data were analyzed using the five tests at each of the nominal alpha levels of .01, .05, and .10. For each test and nominal alpha level, the proportion of the 2000 replications that were significant provided an estimate of the statistical power of each of the tests.

In Table 8 through Table 12, power estimates are reported for $\rho_{13} - \rho_{12} = .2$ when $\rho_{23} = .0$ (Table 8), $\rho_{23} = .2$ (Table 9), $\rho_{23} = .4$ (Table 10), $\rho_{23} = .6$ (Table 11), and $\rho_{23} = .8$ (Table 12). When the value of ρ_{12} was .0 and that of ρ_{13} was .2 at each combination of ρ_{23} and sample size, the five tests had almost the same power. But as the values of ρ_{12} and ρ_{13} increased, the powers of tests T_2, Z_1^*, $\overline{Z}_1^*$, and Z became larger than the power of the proposed test because its power decreased as ρ_{12} increased and the power of the other tests increased as ρ_{12} increased. In general T_2, Z_1^*, $\overline{Z}_1^*$, and Z had very similar power.

In Tables 13 through 17, the power estimates of the five tests are reported for $\rho_{13} - \rho_{12} = .4$ when $\rho_{23} = .0$ (Table 13), $\rho_{23} = .2$ (Table 14), $\rho_{23} = .4$ (Table 15), $\rho_{23} = .6$ (Table 16), and $\rho_{23} = .8$ (Table 17). Trends similar to those found in

Table 8 through Table 12 appear in these five tables. However, there was one exception: The power of the proposed test was similar to the power of the T_2, Z_1^*, $\overline{Z}_1^*$, and Z tests when ρ_{23} was .6 or .8 and sample size was 65 or 100. Overall trends similar to those found in Tables 8 through 12, and Tables 13 through 17, appear in Tables 18 through 22. However, the differences in power between the proposed test and the other tests did not emerge. In Tables 23 through 27, $\rho_{13} - \rho_{12}$ was .8. The five tests had almost identical power.

Tables 8 through Table 27 show the effect on power of an increase in $\rho_{13} - \rho_{12}$ when the value of ρ_{23} is fixed. As expected, when the differences between ρ_{13} and ρ_{12} increases the power of the five tests increases.

In general, comparisons of the following sets, Tables 8 through 12, Tables 13 through 17, Tables 18 through 22, and Tables 23 through 27, indicate that the power of the tests increase as ρ_{23} increases.

Table 3

Estimated Actual Type I Error Rates: $\rho_{12} = \rho_{13} = 0$

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{23} = .0$	.01	30	.009	.009	.010	.008	.006
		65	.008	.008	.009	.007	.007
		100	.010	.010	.011	.009	.009
	.05	30	.052	.052	.056	.051	.045
		65	.045	.047	.048	.045	.044
		100	.044	.045	.046	.044	.043
	.10	30	.099	.101	.104	.100	.098
		65	.099	.102	.103	.100	.098
		100	.093	.094	.094	.094	.091
$\rho_{23} = .2$	.01	30	.007	.009	.009	.006	.005
		65	.010	.010	.010	.010	.008
		100	.009	.009	.008	.008	.008
	.05	30	.046	.045	.048	.044	.039
		65	.049	.049	.049	.048	.046
		100	.041	.043	.043	.041	.041
	.10	30	.092	.095	.095	.092	.087
		65	.095	.095	.097	.095	.093
		100	.095	.095	.095	.095	.094
$\rho_{23} = .4$	.01	30	.014	.014	.014	.012	.008
		65	.013	.013	.012	.012	.015
		100	.013	.013	.013	.011	.011
	.05	30	.056	.056	.057	.052	.049
		65	.054	.054	.054	.052	.051
		100	.054	.054	.054	.053	.052
	.10	30	.108	.109	.108	.105	.101
		65	.111	.109	.112	.109	.108
		100	.110	.110	.110	.110	.109

Table 3--continued

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
ρ_{23} = .6	.01	30	.011	.011	.010	.010	.009
		65	.010	.009	.010	.009	.009
		100	.007	.007	.007	.006	.005+
	.05	30	.048	.049	.047	.045	.042
		65	.048	.050	.047	.046	.045
		100	.057	.056	.057	.056	.055
	.10	30	.098	.099	.098	.093	.089
		65	.097	.095	.097	.096	.094
		100	.107	.107	.107	.107	.107
ρ_{23} = .8	.01	30	.013	.013	.010	.009	.009
		65	.012	.013	.010	.010	.010
		100	.013	.013	.012	.012	.012
	.05	30	.054	.055	.051	.049	.046
		65	.052	.051	.050	.049	.048
		100	.041	.041	.041	.040	.039
	.10	30	.107	.106	.101	.099	.097
		65	.105	.103	.104	.102	.102
		100	.089	.091	.089	.089	.087

* Estimated Type I error rate above $\alpha+2[\alpha(1-\alpha)/2000]^{1/2}$
\+ Estimated Type I error rate below $\alpha-2[\alpha(1-\alpha)/2000]^{1/2}$

Table 4

Estimated Actual Type I Error Rates: $\rho_{12} = \rho_{13} = .2$

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{23} = .0$	.01	30	.011	.008	.012	.009	.008
		65	.010	.008	.012	.010	.008
		100	.007	.006	.008	.007	.007
	.05	30	.050	.042	.054	.051	.048
		65	.058	.052	.061*	.058	.057
		100	.047	.042	.048	.047	.046
	.10	30	.095	.085	.099	.096	.092
		65	.111	.102	.114	.112	.110
		100	.094	.081+	.094	.094	.093
$\rho_{23} = .2$	.01	30	.009	.008	.010	.009	.006
		65	.013	.010	.013	.013	.011
		100	.012	.009	.011	.011	.009
	.05	30	.042	.037+	.047	.042	.036+
		65	.052	.046	.054	.051	.050
		100	.049	.042	.049	.049	.047
	.10	30	.093	.087	.096	.092	.089
		65	.105	.094	.110	.106	.104
		100	.103	.094	.103	.103	.102
$\rho_{23} = .4$	.01	30	.009	.008	.009	.008	.008
		65	.006	.005+	.006	.005+	.004+
		100	.010	.009	.010	.010	.010
	.05	30	.047	.042	.047	.046	.045
		65	.038+	.034+	.040	.038+	.037+
		100	.053	.045	.053	.053	.051
	.10	30	.099	.092	.101	.098	.095
		65	.086+	.079+	.087	.086+	.085+
		100	.105	.098	.106	.105	.104

Table 4--continued

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
ρ_{23} = .6	.01	30	.011	.008	.008	.007	.006
		65	.007	.007	.007	.007	.007
		100	.011	.010	.011	.010	.009
	.05	30	.051	.045	.049	.046	.044
		65	.043	.039+	.043	.042	.042
		100	.050	.044	.050	.050	.049
	.10	30	.098	.092	.098	.097	.094
		65	.096	.089	.096	.095	.094
		100	.092	.087	.092	.092	.091
ρ_{23} = .8	.01	30	.009	.008	.008	.006	.006
		65	.009	.008	.009	.008	.008
		100	.011	.009	.010	.010	.010
	.05	30	.054	.050	.051	.050	.046
		65	.054	.049	.053	.052	.051
		100	.039+	.033+	.038+	.038+	.037+
	.10	30	.098	.091	.096	.095	.094
		65	.101	.093	.101	.098	.098
		100	.096	.086+	.095	.094	.094

* Estimated Type I error rate above $\alpha+2[\alpha(1-\alpha)/2000]^{1/2}$
\+ Estimated Type I error rate below $\alpha-2[\alpha(1-\alpha)/2000]^{1/2}$

Table 5

Estimated Actual Type I Error Rates: $\rho_{12} = \rho_{13} = .4$

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{23} = .0$	.01	30	.011	.005+	.012	.011	.010
		65	.011	.004+	.012	.011	.011
		100	.012	.003+	.013	.013	.012
	.05	30	.051	.026+	.055	.053	.050
		65	.046	.024+	.047	.046	.046
		100	.053	.029+	.055	.054	.053
	.10	30	.092	.063+	.098	.096	.092
		65	.102	.056+	.105	.104	.102
		100	.094	.062+	.095	.095	.094
$\rho_{23} = .2$	.01	30	.012	.004+	.013	.012	.011
		65	.009	.002+	.010	.009	.008
		100	.006	.003+	.006	.006	.006
	.05	30	.044	.024+	.046	.045	.043
		65	.049	.030+	.049	.049	.047
		100	.041	.020+	.044	.042	.041
	.10	30	.098	.063+	.101	.099	.097
		65	.103	.068+	.105	.103	.102
		100	.091	.053+	.092	.091	.091
$\rho_{23} = .4$	.01	30	.012	.005+	.012	.011	.009
		65	.008	.006	.008	.007	.007
		100	.009	.005+	.009	.009	.009
	.05	30	.047	.024+	.048	.046	.045
		65	.052	.028+	.054	.052	.049
		100	.043	.027+	.043	.043	.043
	.10	30	.096	.063+	.101	.096	.092
		65	.102	.067+	.102	.101	.100
		100	.093	.059+	.095	.093	.093

Table 5--continued

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
ρ_{23} = .6	.01	30	.011	.005+	.010	.009	.008
		65	.011	.002+	.011	.009	.008
		100	.009	.004+	.009	.008	.008
	.05	30	.056	.038+	.056	.054	.053
		65	.052	.029+	.053	.051	.050
		100	.045	.025+	.045	.044	.043
	.10	30	.109	.073+	.110	.106	.103
		65	.109	.075+	.109	.108	.106
		100	.096	.059+	.096	.096	.096
ρ_{23} = .8	.01	30	.012	.005+	.011	.009	.009
		65	.010	.005+	.009	.009	.008
		100	.009	.005+	.009	.008	.008
	.05	30	.055	.035+	.054	.052	.050
		65	.050	.032+	.050	.049	.048
		100	.042	.028+	.042	.041	.041
	.10	30	.104	.080+	.102	.100	.100
		65	.101	.067+	.100	.098	.097
		100	.096	.062+	.095	.094	.094

* Estimated Type I error rate above $\alpha+2[\alpha(1-\alpha)/2000]^{1/2}$
\+ Estimated Type I error rate below $\alpha-2[\alpha(1-\alpha)/2000]^{1/2}$

Table 6

Estimated Actual Type I Error Rates: $\rho_{12} = \rho_{13} = .6$

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{23} = .0$	.01	30	.007	.000$^+$	.012	.011	.012
		65	.010	.001$^+$	.010	.010	.010
		100	.010	.001$^+$	.010	.010	.010
	.05	30	.058	.004$^+$	.061*	.060	.060
		65	.035$^+$	.006$^+$	.036$^+$	.036$^+$	.036$^+$
		100	.046	.006$^+$	.047	.046	.046
	.10	30	.101	.020$^+$	.103	.102	.103
		65	.087	.014$^+$	.088	.088	.088
		100	.010	.017$^+$	.097	.096	.096
$\rho_{23} = .2$	.01	30	.005$^+$	.000$^+$	.009	.007	.007
		65	.009	.001$^+$	.011	.010	.009
		100	.013	.002$^+$	.014	.013	.013
	.05	30	.045	.005$^+$	.050	.049	.047
		65	.047	.007$^+$	.048	.048	.047
		100	.052	.010$^+$	.052	.052	.052
	.10	30	.099	.021$^+$	.105	.103	.102
		65	.086$^+$	.021$^+$	.087	.087	.087
		100	.091	.027$^+$	.092	.092	.092
$\rho_{23} = .4$	.01	30	.007	.001$^+$	.009	.008	.007
		65	.012	.001$^+$	.013	.012	.012
		100	.007	.001$^+$	.007	.007	.007
	.05	30	.045	.007$^+$	.049	.048	.047
		65	.053	.010$^+$	.054	.053	.053
		100	.042	.006$^+$	.043	.043	.043
	.10	30	.098	.021$^+$	.101	.100	.100
		65	.102	.026$^+$	.104	.103	.102
		100	.101	.017$^+$	.102	.102	.102

Table 6--continued

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
ρ_{23} = .6	.01	30	.006	.000+	.006	.006	.005+
		65	.010	.001+	.010	.010	.010
		100	.009	.001+	.009	.008	.008
	.05	30	.047	.008+	.050	.046	.045
		65	.058	.009+	.059	.058	.058
		100	.050	.009+	.051	.050	.050
	.10	30	.098	.028+	.100	.098	.097
		65	.110	.032+	.112	.111	.110
		100	.101	.030+	.101	.101	.101
ρ_{23} = .8	.01	30	.011	.003+	.012	.010	.010
		65	.012	.002+	.013	.012	.012
		100	.011	.002+	.011	.011	.011
	.05	30	.053	.015+	.055	.052	.051
		65	.058	.013+	.059	.058	.057
		100	.051	.014+	.051	.050	.049
	.10	30	.101	.042+	.103	.100	.099
		65	.108	.040+	.108	.107	.107
		100	.114	.040+	.114	.114	.113

* Estimated Type I error rate above $\alpha+2[\alpha(1-\alpha)/2000]^{1/2}$
\+ Estimated Type I error rate below $\alpha-2[\alpha(1-\alpha)/2000]^{1/2}$

Table 7

Estimated Actual Type I Error Rates: $\rho_{12} = \rho_{13} = .8$

Value of ρ_{23}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{23} = .4$	.01	30	.007	.000+	.009	.009	.009
		65	.013	.000+	.013	.013	.013
		100	.013	.000+	.015*	.014	.015*
	.05	30	.047	.000+	.052	.050	.051
		65	.056	.000+	.057	.056	.057
		100	.045	.000+	.046	.046	.046
	.10	30	.102	.001+	.106	.105	.106
		65	.100	.000+	.102	.101	.102
		100	.100	.000+	.101	.101	.101
$\rho_{23} = .6$	.01	30	.012	.000+	.015*	.015*	.015*
		65	.011	.000+	.012	.011	.011
		100	.011	.000+	.012	.012	.012
	.05	30	.042	.000+	.047	.047	.047
		65	.047	.000+	.050	.050	.050
		100	.048	.000+	.049	.049	.049
	.10	30	.087	.002+	.093	.093	.093
		65	.100	.002+	.101	.101	.101
		100	.097	.002+	.099	.099	.098
$\rho_{23} = .8$	.01	30	.007	.000+	.009	.007	.006
		65	.012	.000+	.012	.012	.012
		100	.008	.000+	.009	.008	.008
	.05	30	.041	.001+	.043	.041	.040
		65	.057	.001+	.059	.058	.057
		100	.051	.001+	.052	.051	.051
	.10	30	.089	.004+	.096	.092	.092
		65	.111	.003+	.113	.111	.111
		100	.095	.003+	.097	.096	.096

* Estimated Type I error rate above $\alpha+2[\alpha(1-\alpha)/2000]^{1/2}$
\+ Estimated Type I error rate below $\alpha-2[\alpha(1-\alpha)/2000]^{1/2}$

Table 8

Power of the Five Tests: $\rho_{13} = \rho_{12} = .2$ and $\rho_{23} = .0$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.033	.032	.036	.031	.025
		65	.074	.071	.076	.071	.067
		100	.113	.109	.114	.112	.108
	.05	30	.108	.110	.116	.108	.104
		65	.203	.200	.207	.203	.200
		100	.295	.289	.296	.295	.290
	.10	30	.185	.183	.189	.185	.178
		65	.310	.305	.314	.310	.305
		100	.416	.414	.420	.417	.414
$\rho_{12} = .2$	.01	30	.039	.027	.046	.039	.034
		65	.080	.059	.083	.080	.077
		100	.137	.099	.140	.137	.133
	.05	30	.142	.109	.153	.144	.139
		65	.230	.185	.234	.230	.227
		100	.330	.276	.333	.331	.327
	.10	30	.234	.197	.239	.235	.229
		65	.335	.287	.337	.336	.335
		100	.444	.392	.446	.445	.443
$\rho_{12} = .4$	.01	30	.051	.010	.059	.056	.053
		65	.109	.034	.116	.113	.111
		100	.189	.063	.194	.192	.190
	.05	30	.153	.067	.161	.158	.157
		65	.290	.141	.294	.291	.291
		100	.419	.220	.423	.421	.419
	.10	30	.248	.131	.254	.252	.251
		65	.423	.246	.429	.428	.426
		100	.553	.374	.554	.554	.553
$\rho_{12} = .6$	.01	30	.111	.001	.133	.120	.134
		65	.342	.003	.362	.352	.363
		100	.579	.007	.593	.585	.594
	.05	30	.303	.014	.312	.302	.315
		65	.619	.054	.624	.618	.624
		100	.807	.129	.808	.806	.808
	.10	30	.434	.040	.436	.430	.436
		65	.745	.154	.745	.744	.745
		100	.887	.307	.887	.886	.887

Table 9

Power of the Five Tests: $\rho_{13} - \rho_{12} = .2$ and $\rho_{23} = .2$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.040	.039	.042	.036	.028
		65	.083	.082	.085	.080	.077
		100	.170	.167	.172	.166	.159
	.05	30	.140	.142	.144	.137	.129
		65	.248	.241	.249	.246	.239
		100	.356	.357	.359	.356	.352
	.10	30	.221	.224	.229	.220	.216
		65	.370	.365	.372	.369	.363
		100	.484	.480	.486	.485	.481
$\rho_{12} = .2$	.01	30	.045	.031	.048	.044	.038
		65	.109	.082	.114	.108	.103
		100	.408	.360	.411	.408	.404
	.05	30	.141	.115	.147	.140	.136
		65	.263	.225	.226	.262	.260
		100	.408	.360	.411	.408	.404
	.10	30	.237	.197	.243	.235	.234
		65	.370	.329	.374	.370	.336
		100	.536	.491	.538	.536	.536
$\rho_{12} = .4$	.01	30	.049	.012	.056	.053	.048
		65	.152	.051	.160	.155	.152
		100	.265	.109	.269	.266	.262
	.05	30	.176	.067	.181	.178	.175
		65	.347	.188	.350	.349	.348
		100	.475	.306	.481	.479	.474
	.10	30	.280	.150	.289	.287	.282
		65	.470	.317	.473	.472	.471
		100	.609	.446	.612	.610	.609
$\rho_{12} = .6$	.01	30	.112	.002	.138	.130	.134
		65	.372	.011	.386	.382	.386
		100	.610	.037	.625	.621	.624
	.05	30	.302	.020	.317	.312	.316
		65	.622	.098	.627	.624	.627
		100	.825	.235	.828	.827	.828
	.10	30	.441	.066	.447	.443	.445
		65	.743	.234	.745	.744	.745
		100	.897	.439	.898	.897	.897

Table 10

Power of the Five Tests: $\rho_{13} - \rho_{12} = .2$ and $\rho_{23} = .4$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.051	.052	.052	.044	.038
		65	.143	.142	.142	.135	.129
		100	.217	.214	.216	.211	.208
	.05	30	.165	.162	.168	.161	.153
		65	.316	.313	.316	.314	.310
		100	.445	.439	.445	.442	.438
	.10	30	.247	.244	.251	.246	.240
		65	.438	.433	.440	.436	.432
		100	.565	.559	.567	.563	.560
$\rho_{12} = .2$	.01	30	.063	.043	.063	.055	.050
		65	.146	.110	.147	.139	.134
		100	.250	.197	.251	.245	.240
	.05	30	.183	.148	.186	.178	.171
		65	.332	.287	.335	.330	.327
		100	.486	.430	.487	.485	.481
	.10	30	.277	.245	.281	.276	.270
		65	.472	.423	.474	.470	.468
		100	.608	.564	.609	.608	.605
$\rho_{12} = .4$	.01	30	.075	.025	.078	.071	.065
		65	.201	.079	.206	.201	.197
		100	.361	.171	.364	.361	.355
	.05	30	.210	.103	.217	.210	.200
		65	.417	.260	.424	.418	.413
		100	.606	.430	.609	.607	.603
	.10	30	.330	.195	.336	.331	.326
		65	.536	.398	.539	.538	.536
		100	.718	.587	.719	.718	.718
$\rho_{12} = .6$	.01	30	.160	.006	.180	.175	.173
		65	.452	.028	.464	.460	.458
		100	.688	.105	.695	.693	.692
	.05	30	.379	.051	.393	.392	.391
		65	.704	.190	.710	.709	.708
		100	.873	.393	.876	.875	.875
	.10	30	.517	.125	.527	.524	.523
		65	.810	.380	.813	.812	.812
		100	.925	.605	.926	.926	.926

Table 11

Power of the Five Tests: $\rho_{13} - \rho_{12} = .2$ and $\rho_{23} = .6$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.077	.076	.075	.066	.058
		65	.215	.212	.206	.198	.194
		100	.383	.376	.379	.375	.368
	.05	30	.233	.231	.229	.221	.216
		65	.433	.426	.431	.429	.426
		100	.629	.626	.629	.627	.624
	.10	30	.347	.343	.348	.341	.335
		65	.558	.557	.558	.557	.555
		100	.748	.745	.748	.747	.747
$\rho_{12} = .2$	.01	30	.090	.066	.086	.075	.069
		65	.251	.207	.247	.240	.233
		100	.440	.384	.437	.433	.430
	.05	30	.240	.209	.242	.233	.227
		65	.472	.426	.471	.467	.462
		100	.677	.627	.676	.672	.669
	.10	30	.353	.312	.357	.349	.345
		65	.595	.551	.597	.594	.593
		100	.778	.751	.778	.778	.778
$\rho_{12} = .4$	.01	30	.109	.034	.114	.098	.089
		65	.341	.175	.342	.333	.327
		100	.525	.322	.527	.523	.517
	.05	30	.287	.161	.291	.280	.272
		65	.599	.438	.602	.596	.592
		100	.778	.610	.778	.776	.773
	.10	30	.407	.276	.409	.405	.403
		65	.733	.597	.736	.732	.730
		100	.852	.767	.853	.852	.850
$\rho_{12} = .6$	.01	30	.235	.011	.254	.246	.239
		65	.604	.108	.614	.608	.606
		100	.865	.305	.868	.866	.865
	.05	30	.465	.112	.480	.472	.469
		65	.821	.387	.823	.822	.821
		100	.952	.681	.953	.952	.952
	.10	30	.593	.226	.598	.596	.595
		65	.892	.580	.893	.892	.892
		100	.973	.855	.974	.974	.973

Table 12

Power of the Five Tests: $\rho_{13} - \rho_{12} = .2$ and $\rho_{23} = .8$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.189	.180	.170	.161	.154
		65	.483	.477	.465	.460	.456
		100	.736	.731	.728	.727	.725
	.05	30	.393	.388	.383	.374	.370
		65	.735	.729	.730	.728	.725
		100	.895	.894	.894	.894	.894
	.10	30	.527	.525	.522	.519	.515
		65	.826	.825	.824	.823	.821
		100	.940	.937	.940	.940	.940
$\rho_{12} = .2$	.01	30	.215	.179	.194	.182	.175
		65	.549	.483	.539	.526	.519
		100	.797	.745	.790	.788	.785
	.05	30	.440	.391	.431	.421	.415
		65	.774	.735	.770	.769	.767
		100	.931	.916	.930	.929	.929
	.10	30	.569	.529	.567	.562	.560
		65	.863	.829	.861	.860	.860
		100	.961	.953	.961	.961	.961
$\rho_{12} = .4$	.01	30	.269	.138	.260	.237	.229
		65	.683	.501	.682	.671	.665
		100	.893	.759	.892	.888	.887
	.05	30	.516	.367	.515	.504	.495
		65	.860	.762	.860	.857	.856
		100	.960	.923	.960	.960	.960
	.10	30	.646	.514	.648	.641	.639
		65	.923	.858	.923	.923	.922
		100	.980	.962	.980	.980	.980
$\rho_{12} = .6$	.01	30	.524	.103	.539	.505	.494
		65	.914	.488	.914	.910	.909
		100	.989	.821	.990	.989	.989
	.05	30	.755	.343	.767	.750	.747
		65	.975	.820	.976	.975	.975
		100	.999	.969	.999	.999	.999
	.10	30	.848	.532	.854	.849	.848
		65	.987	.914	.987	.987	.987
		100	.999	.988	.999	.999	.999

Table 13

Power of the Five Tests: $\rho_{13} - \rho_{12} = .4$ and $\rho_{23} = .0$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.151	.132	.160	.144	.132
		65	.393	.365	.401	.391	.380
		100	.668	.637	.676	.665	.659
	.05	30	.351	.329	.362	.351	.339
		65	.639	.618	.645	.639	.635
		100	.853	.838	.854	.853	.852
	.10	30	.482	.464	.490	.486	.476
		65	.761	.736	.764	.763	.759
		100	.910	.902	.911	.910	.909
$\rho_{12} = .2$	.01	30	.181	.102	.207	.109	.176
		65	.525	.371	.535	.528	.521
		100	.774	.623	.779	.775	.773
	.05	30	.401	.205	.414	.409	.402
		65	.764	.648	.767	.765	.763
		100	.915	.851	.917	.916	.915
	.10	30	.539	.413	.550	.545	.542
		65	.848	.776	.851	.849	.848
		100	.960	.928	.961	.960	.960
$\rho_{12} = .4$	.01	30	.394	.051	.445	.433	.441
		65	.823	.320	.834	.829	.833
		100	.973	.660	.975	.975	.975
	.05	30	.680	.244	.694	.688	.692
		65	.948	.676	.949	.948	.949
		100	.994	.930	.994	.994	.994
	.10	30	.793	.428	.800	.797	.798
		65	.976	.817	.977	.977	.977
		100	.998	.973	.998	.998	.998

Table 14

Power of the Five Tests: $\rho_{13} - \rho_{12} = .4$ and $\rho_{23} = .2$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.189	.169	.169	.177	.151
		65	.531	.497	.536	.524	.510
		100	.779	.759	.781	.774	.771
	.05	30	.419	.400	.424	.413	.404
		65	.759	.740	.762	.759	.752
		100	.915	.906	.915	.914	.912
	.10	30	.536	.517	.544	.536	.527
		65	.846	.827	.848	.845	.843
		100	.946	.938	.946	.946	.944
$\rho_{12} = .2$	.01	30	.252	.155	.263	.251	.229
		65	.661	.500	.672	.660	.650
		100	.880	.796	.883	.878	.875
	.05	30	.503	.383	.516	.506	.495
		65	.852	.769	.855	.853	.850
		100	.962	.928	.963	.962	.961
	.10	30	.640	.540	.645	.643	.634
		65	.918	.869	.919	.919	.918
		100	.980	.968	.980	.980	.980
$\rho_{12} = .4$	.01	30	.459	.094	.490	.481	.478
		65	.905	.502	.909	.909	.909
		100	.988	.836	.990	.989	.989
	.05	30	.711	.341	.723	.721	.720
		65	.971	.836	.971	.971	.971
		100	.998	.969	.998	.998	.998
	.10	30	.804	.517	.812	.811	.809
		65	.984	.923	.985	.985	.984
		100	.999	.990	.999	.999	.999

Table 15

Power of the Five Tests: $\rho_{13} - \rho_{12} = .4$ and $\rho_{23} = .4$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.299	.278	.299	.282	.260
		65	.686	.655	.685	.673	.663
		100	.911	.893	.912	.909	.906
	.05	30	.550	.524	.555	.542	.526
		65	.871	.864	.872	.870	.868
		100	.975	.969	.975	.974	.972
	.10	30	.676	.661	.680	.674	.669
		65	.934	.926	.934	.933	.933
		100	.989	.986	.989	.989	.989
$\rho_{12} = .2$	.01	30	.361	.223	.369	.347	.319
		65	.804	.687	.806	.802	.793
		100	.951	.903	.951	.949	.949
	.05	30	.632	.520	.638	.629	.620
		65	.934	.982	.934	.934	.931
		100	.987	.975	.988	.987	.987
	.10	30	.743	.663	.748	.741	.738
		65	.966	.944	.967	.966	.966
		100	.995	.990	.994	.995	.995
$\rho_{12} = .4$	.01	30	.611	.215	.644	.628	.613
		65	.962	.724	.964	.963	.962
		100	.997	.949	.998	.998	.997
	.05	30	.835	.519	.842	.838	.836
		65	.994	.929	.994	.994	.994
		100	1.000	.997	1.000	1.000	1.000
	.10	30	.899	.694	.904	.904	.902
		65	1.000	.974	1.000	1.000	1.000
		100	1.000	.999	1.000	1.000	1.000

Table 16

Power of the Five Tests: $\rho_{13} - \rho_{12} = .4$ and $\rho_{23} = .6$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.470	.441	.452	.431	.401
		65	.910	.895	.905	.902	.899
		100	.989	.987	.989	.988	.988
	.05	30	.724	.704	.721	.714	.707
		65	.974	.972	.973	.972	.972
		100	.998	.998	.998	.998	.998
	.10	30	.822	.812	.821	.816	.814
		65	.991	.989	.991	.990	.990
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .2$	.01	30	.612	.470	.608	.588	.560
		65	.954	.901	.954	.952	.951
		100	.996	.992	.996	.996	.995
	.05	30	.829	.745	.829	.821	.814
		65	.993	.981	.993	.993	.993
		100	1.000	.999	1.000	1.000	1.000
	.10	30	.896	.854	.896	.896	.893
		65	.996	.993	.996	.995	.995
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .4$	.01	30	.842	.438	.848	.837	.830
		65	.999	.949	.999	.999	.998
		100	1.000	.998	1.000	1.000	1.000
	.05	30	.945	.779	.947	.945	.942
		65	1.000	.995	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.977	.892	.978	.978	.976
		65	1.000	.999	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 17

Power of the Five Tests: $\rho_{13} - \rho_{12} = .4$ and $\rho_{23} = .8$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.919	.899	.898	.890	.886
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.977	.971	.975	.973	.973
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.991	.989	.990	.990	.990
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .2$	.01	30	.969	.922	.967	.962	.960
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.995	.988	.995	.995	.995
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.998	.995	.998	.997	.997
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .4$	.01	30	1.000	.951	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	1.000	.995	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	.999	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 18

Power of the Five Tests: $\rho_{13} - \rho_{12} = .6$ and $\rho_{23} = .0$ (fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.486	.408	.501	.485	.463
		65	.910	.863	.916	.910	.905
		100	.989	.980	.990	.989	.989
	.05	30	.728	.662	.740	.730	.723
		65	.977	.966	.979	.978	.976
		100	.998	.997	.998	.998	.998
	.10	30	.832	.786	.837	.833	.828
		65	.988	.985	.988	.988	.988
		100	.999	.998	.999	.999	.999
$\rho_{12} = .2$	.01	30	.728	.396	.761	.751	.746
		65	.986	.904	.986	.986	.986
		100	1.000	.996	1.000	1.000	1.000
	.05	30	.907	.715	.914	.911	.910
		65	.998	.980	.998	.998	.998
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.949	.852	.951	.951	.950
		65	.999	.995	.999	.999	.999
		100	1.000	1.000	1.000	1.000	1.000

Table 19

Power of the Five Tests: $\rho_{13} - \rho_{12} = .6$ and $\rho_{23} = .2$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.623	.534	.640	.615	.587
		65	.970	.946	.970	.969	.967
		100	.999	.997	.999	.999	.999
	.05	30	.838	.789	.841	.831	.831
		65	.992	.987	.993	.992	.991
		100	1.000	1.000	.1000	1.000	1.000
	.10	30	.906	.879	.908	.906	.905
		65	.997	.996	.997	.997	.997
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .2$	.01	30	.827	.527	.844	.834	.824
		65	.997	.975	.997	.997	.997
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.944	.828	.947	.945	.943
		65	1.000	.997	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.974	.914	.975	.974	.974
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 20

Power of the Five Tests: $\rho_{13} - \rho_{12} = .6$ and $\rho_{23} = .4$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.809	.742	.810	.794	.779
		65	.997	.994	.997	.997	.996
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.935	.903	.937	.933	.932
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.968	.955	.969	.967	.965
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .2$	.01	30	.954	.782	.958	.952	.947
		65	1.000	.999	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.989	.946	.990	.990	.989
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.994	.983	.994	.994	.994
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 21

Power of the Five Tests: $\rho_{13} - \rho_{12} = .6$ and $\rho_{23} = .6$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.978	.963	.977	.974	.969
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.998	.995	.998	.998	.997
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .2$	.01	30	1.000	.983	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 22

Power of the Five Tests: $\rho_{13} - \rho_{12} = .6$ and $\rho_{23} = .8$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
$\rho_{12} = .2$	.01	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 23

Power of the Five Tests: $\rho_{13} - \rho_{12} = .8$ and $\rho_{23} = .0$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.939	.812	.944	.942	.939
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.985	.956	.985	.985	.984
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.991	.982	.991	.991	.991
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 24

Power of the Five Tests: $\rho_{13} - \rho_{12} = .8$ and $\rho_{23} = .2$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	.980	.932	.980	.980	.978
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	.996	.981	.996	.996	.996
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	.999	.994	.999	.999	.999
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 25

Power of the Five Tests: $\rho_{13} - \rho_{12} = .8$ and $\rho_{23} = .4$ (fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	1.000	.998	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 26

Power of the Five Tests: $\rho_{13} - \rho_{12} = .8$ and $\rho_{23} = .6$(fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

Table 27

Power of the Five Tests: $\rho_{13} - \rho_{12} = .8$ and $\rho_{23} = .8$ (fixed)

Value of ρ_{12}	Nominal Alpha	Sample Size	T_2	P	Z_1^*	$\overline{Z}_1^*$	Z
$\rho_{12} = .0$	.01	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.05	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000
	.10	30	1.000	1.000	1.000	1.000	1.000
		65	1.000	1.000	1.000	1.000	1.000
		100	1.000	1.000	1.000	1.000	1.000

CHAPTER 4

CONCLUSIONS

Important considerations for a researcher who must choose among various statistical procedures for testing equality of dependent correlation coefficients are the Type I error rates and statistical power of the various alternative procedures. Decisions regarding the choice of a statistical procedure should be made on the basis of such considerations.

In this study, the simple t-test on a single correlation was examined to find if it can be adapted for testing the null hypothesis H_o: $\rho_{12} = \rho_{13}$. Using simulated data the appropriateness of this proposed procedure was evaluated and compared to Williams' procedure (T_2) and three procedures (Z_1^*, $\overline{Z}_1^*$, and Z) based on Fisher's z transformation. For the proposed test a number of discrepancies between nominal and estimated actual alpha levels were observed for the 23 experimental conditions in which $\rho_{12} = \rho_{13}$. However, the other tests had appropriate and very similar estimated actual Type I error rates. These results did not support Neil and Dunn's finding in which Z_1^* procedure has high estimated Type I error rates.

Comparisons of the statistical power of the five procedures were made under the various combinations of ρ_{12}, ρ_{13}, and ρ_{23}. The results of those comparisons have been reported in

Tables 8 through 27. When $\rho_{13} - \rho_{12} = .2$, as the values of ρ_{12} and ρ_{13} increased with the value of ρ_{23} fixed, the power of tests T_2, Z_1^*, $\overline{Z}_1^*$, and Z became larger than the power of the proposed test. However, the proposed test had almost the same power as that of the remaining tests when the values of $\rho_{13} - \rho_{12}$ and ρ_{23} increased to .6 and .8. In general, when the value of ρ_{23} and the difference between ρ_{13} and ρ_{12} increased, the power of the five tests increased. Typically, T_2, Z_1^*, $\overline{Z}_1^*$, and Z tests had very similar power. However, Neil and Dunn found that T_2 has more power than Z_1^*. Their finding was considered as an unexpected result because Fisher's z transformation of sample correlation coefficients improves normality substantially, especially for small sample sizes and extreme sample correlations. As a result statistics based on $Z_{12} - Z_{13}$ should be preferred to those based on $r_{12} - r_{13}$.

When testing the null hypothesis of the equality of correlation coefficients, empirical evidence suggests that the proposed procedure should not be used. The remaining procedures have almost the same Type I error rates and power under each condition. So the choice among four procedures (T_2, Z_1^*, $\overline{Z}_1^*$, Z) is inconsequential for the conditions included in this study.

There are three limitations to the present study. First, only positive correlation combinations were included in the experimental conditions. Second, sample sizes smaller than 35 were not studied. Third only multivariate normal data were

simulated. Using combinations of positive and negative correlation coefficients, smaller sample sizes, and/or nonnormal data might result in different conclusions regarding the importance of choice of a statistical procedure. Given these limitations, the results of this study provide useful information for the researcher who must choose among the T_2, P, Z_1^*, $\overline{Z}_1^*$, and Z procedures for use in conditions similar to those used in this study.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a thesis for the degree of Master of Arts in Education.

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